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Research Article

Solution Approach to Cutting Stock Problems Using Iterative Trim Loss Algorithm and Monte-Carlo Simulation

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ABSTRACT

Cutting Stock Problems are the most studied NP-Hard combinatorial problems among optimization problems. A One-dimensional Cutting Stock Problem (1-CSP), which aims to create cutting patterns to minimize trim loss, is one of the best known optimization problems. The difficulty of the solution stages and the lack of a definite solution method that can be applied to all problems have caused these problems to attract a lot of attention by researchers. This study includes a hybrid solution algorithm combined with iterative trim loss algorithm and Monte Carlo simulations, and a comparative study of the method with the solution methods in the literature, for the solution of orders to be obtained with minimum cutting loss from the same type of stocks.

Keywords: Combinatorial Optimization, NP-Hard, Cutting Stock Problems

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1. Introduction

Productivity and high performance are crucial for the success of any organization, but many critical tasks involve complex optimization problems that are difficult to solve. To achieve better outcomes, these problems must be addressed using computational tools that can adapt to different scenarios and obtain good solutions without consuming excessive resources. Optimization problems can be expressed mathematically, but they are typically challenging to solve. These problems involve finding values of variables that optimize a function, called the objective function, while satisfying constraints on the output variables. Optimization problems are fundamental to many fields, including operations research, engineering, economics, and computer science, and can be classified into different classes based on the types of variables and objective functions involved.

Combinatorial optimization problems, such as the one-dimensional cutting stock problem (1D-CSP), involve finding the optimal combination of discrete objects subject to certain constraints. The 1D-CSP is a challenging problem that arises in various industrial settings, such as paper production, steel manufacturing, and woodworking. In this problem, a single-dimensional stock material must be cut into smaller pieces to meet customer demand while minimizing waste.

The one-dimensional cutting stock problem was first discussed by Kantorovich in 1960 as a type of cutting and packing problem. Dyckoff (1990) defined a classic 1D-CSP as a problem related to the supply of a certain number of small objects of the same or different lengths from an infinite number of large objects that can have various lengths. Similarly, these problems are classified as NP-hard problems in the works of Kantorovich (1960) and Gilmore and Gomory (1961). The 1D-CSP is encountered in many industries worldwide, such as shipbuilding, forestry, rubber molding, and aluminum door production, among others.

In practical applications, the one-dimensional cutting stock problem requires efficient use of raw materials. For example, in the shipbuilding industry, the problem involves cutting metal sheets of different sizes into smaller pieces to fit the various parts of the ship. Similarly, in the forestry industry, cutting logs of varying lengths into smaller pieces to fit the demands of different customers is a common scenario. Despite the development of efficient algorithms, the problem remains challenging due to its combinatorial nature and the large number of variables involved.

The main objective of classical 1D-CSP is to minimize cutting losses, as described in Alfares and Alsawafy (2019) and Evtimov and Fidanova (2018). While the theoretical problem can be described in a sufficiently simple manner, many constraints can be added to the problem according to industry requirements and demands. For example, Peng and Chu (2010a) and (Peng and Chu, 2010b) worked on minimizing stock numbers in their studies, while Machado et al. (2020) and Ravelo et al. (2020) focused on obtaining cutting losses to be reused in their studies.

During the 1950s, several studies explored the topic of cutting stock problems. However, it was not until 1960, when Kantorovic applied linear programming to 1D-CSP, and Gilmore and Gomory (1961 and 1963) introduced column generation techniques, that the first studies applicable to real-life problems in this field emerged. Over the years, researchers have developed various approaches to minimize cutting losses. Among these are genetic algorithms proposed by Chen et al. (2019) and Hinterding and Khan (1993), as well as evolutionary programming (EP) approaches proposed by Liang et al. (2002). In their proposal, Liang et al. suggested an EP solution approach to minimize waste and stock usage. They proposed using the item list as a vector and representing a chromosome that would generate item arrays in the mutation process according to the length of the object. Meanwhile, Levine and Ducatelle (2004) suggested that the pure Ant Colony Optimization (ACO) approach could compete with existing evolutionary methods. They also proposed a hybrid approach that enriched the ACO with a local search algorithm, which could outperform the best-known hybrid evolutionary solution methods for certain problem classes.

In this study, we propose a hybrid algorithm that combines the iterative trim loss algorithm with Monte Carlo simulations to minimize cutting loss for orders that require the same type of stock. Our solution method achieves this goal with a reasonable computational effort. While the iterative trim loss algorithm requires complete information about the surrounding or all possible transitions, Monte Carlo methods work on a state-action trajectory sampled on a portion. Furthermore, the iterative trim loss algorithm considers only a one-step transition, whereas Monte Carlo goes all the way to the terminal node. To leverage the strengths of both methods, we adopt a solution algorithm that starts with an initial solution obtained through iterative linear programming and improves it with the Monte Carlo method. This method assumes each situation's solution independently, allowing for a more efficient optimization process.

The iterative trim loss algorithm is a mathematical optimization technique used in cutting stock problems. It requires complete information about the surrounding or all possible transitions, and only a one-step transition is considered. The algorithm minimizes the cutting loss in the cutting plan while satisfying the demand.

On the other hand, Monte Carlo simulation is a statistical method that uses random sampling to obtain numerical solutions to problems. It works on a state-action trajectory sampled on a portion, going all the way to the terminal node. The required lengths are weighted according to the part lengths, and different cutting plans are requested.

The hybrid algorithm combines the strengths of both methods, taking into account the iterative trim loss algorithm's strength in minimizing cutting loss while satisfying the demand and Monte Carlo's strength in generating random solutions to improve the initial solution obtained with iterative linear programming. By doing so, the algorithm aims to achieve minimum cutting loss and reduce waste, which is crucial for efficient and sustainable manufacturing processes. Overall, this approach represents a novel and promising solution for optimizing cutting plans that can be applied to various industries, including woodworking, metalworking, and plastic manufacturing.

2. Mathematical Model

The one-dimensional cutting stock problem is a combinatorial optimization problem that involves finding the optimal way to cut one-dimensional materials such as rolls, pipes, or sheets into smaller pieces of specified lengths with minimal waste. The classical version of this problem assumes that there is a single type of material that is available in rolls or sheets of a fixed length, and that there is a set of demand for smaller pieces of different lengths that need to be cut from the available material.

The mathematical model for the classical one-dimensional cutting stock problem can be formulated as follows:

Let L denote the set of orders, with index i, and B denote the set of stock items, with index j. The parameters L_i and L represent the length of the ith order and the length of each stock item, respectively. The decision variables are $x_{i,b}$ which is 1 if order i is cut from stock j, and 0 otherwise, and y_b, which is 1 if stock j is used, and 0 otherwise

The objective is to minimize the total number of used stocks, which can be expressed as:

$$Min \sum_{b \in B} y_b$$

The problem is subject to the following constraints:

$$\sum_{b \in B} x_{ib} = 1 \qquad \forall i \in I \tag{1}$$

$$\sum_{i \in I} L_i x_{ib} \leq L y_b \quad \forall b \in B \tag{2}$$

$$x_{ib} \leq y_b \qquad \forall i \in I, b \in B \tag{3}$$

$$x_{ib} \in \{0,1\} \qquad \forall i \in I, b \in B \tag{4}$$

$$y_b \in \{0,1\} \qquad \forall b \in B \tag{5}$$

3. Methodology

The main motivation of this study is to systematize and improve the manually created cutting plans as much as possible by using algorithmic solutions. In this regard, a cutting stock problem solution is modeled by grouping the properties from the Bin Packing Problem (BPP) and the Generalized Assignment Problem (GAP). The solution to the problem consists of two stages. The first stage of the method involves the application of the iterative trimm loss algorithm, which was inspired by the logical explanation put forth by Falkenauer and Delchambre (1992) in their seminal study on assembly line balancing and bin packing problems. In their study, Falkenauer and Delchambre showed that focusing on the fullness of the relevant boxes is better than focusing on the overall performance of all the boxes. They explained this with the example that "if we take two boxes and mix their contents, it is better for one box to be almost full (leaving the other almost empty) than for the two boxes to be filled approximately equally. The reason for this is that the almost empty compartment can more easily accommodate additional objects that would fit very easily into any of the normally half-full compartments." In this sense, the problem-solving approach is determined as a solution stage similar to the placement algorithms applied in bin packing problems, which focuses on obtaining the cutting plan with the least waste from the relevant stock rather than the overall performance of all stocks. In the first step, one stock is included in the problem. The objective function is defined as zero waste, and a linear integer programming approach is used considering the order quantities. Then, the orders obtained by cutting from the first stock are subtracted from the total orders, and the second step is started by including another stock. This process is repeated until the step where a waste-free assignment cannot be made. At the step where a waste-free solution cannot be found, the problem is included by defining the waste as minimal. Thus, at each step, an algorithmic solution method is adopted to reach an appropriate solution. In the first stage, as the variables involved in the problem will change and solving the problem step-by-step would be time-consuming, a calculation tool was developed using Visual Basic for Applications (VBA) and the Opensolver reference, which can solve optimization problems automatically.

In the second stage of the method, the cutting plan is considered based on the appropriate solution obtained in the first stage, and the aim is to improve the problem solution, if possible. To achieve this, a Monte Carlo simulation is used, which is implemented through a VBA code. The simulation randomly selects the number of stocks that give errors from the stocks that do not give errors in the cutting plan, and creates different cutting plans by weighting the required lengths according to the part lengths. This process is repeated for a specified number of iterations, and the solution that gives the minimum error amount is selected from among all solutions as the final solution at the end of the iteration. By using this approach, the cutting plan can be optimized further, thus minimizing waste and improving efficiency.

Figure 1. Flow chart showing problem solving steps

The proposed problem solving algorithm is depicted in the flowchart below, which provides a clear visual representation of the iterative trimm loss and Monte Carlo simulation techniques used in this study to obtain an optimized cutting plan with minimum waste. The flowchart shows the step-by-step process of solving the cutting stock problem, starting from the definition of the objective function, through the inclusion of each stock in turn, until the final cutting plan is obtained.

Our method is designed to produce cutting plans with minimum waste. By automating the problem-solving process and improving the cutting plan, we provide a highly efficient solution for the optimization problem. We developed a calculation tool in VBA with the Opensolver reference to obtain results with the minimum waste amount for a large number of cutting plans. Additionally, we use Monte Carlo simulation in the second stage to select the best cutting plan from a set of potential solutions, increasing the accuracy and reliability of the final result. Our approach is an effective solution for optimizing the cutting plan and reducing waste, providing significant benefits for manufacturing and production processes.

4. Data Set and Comparative Study

To test the effectiveness of the proposed solution method, 10 different sample datasets were used. The first five datasets (1a-5a) were obtained from Hinterding and Khan's studies, while the last five were obtained from Liang et al.'s studies. The aim of these sample datasets was to obtain minimum waste while fulfilling orders for the same types of stock lengths.

The information regarding the sample dataset is summarized in the table below:

Problem	Number of Product Types	Order Quantity	Stock Length
1a	8	20	14
2a	8	50	15
3a	8	60	25
4a	8	60	25
5a	18	126	4300
6a	18	200	86
7a	24	200	120
8a	24	400	120
9a	36	400	120
10a	36	600	120

Table 1. The Characteristics of the Problem

Tanır et al.(2018) used the same dataset in their study to test the effectiveness of the proposed method. The results obtained from the proposed solution method, along with those obtained from the studies on the intuitive method developed by Tanir et al., evolutionary programming, and the commercial software Real Cut 1D applied to the same problems, are presented in the table below. The table 2 summarizes the results of the methods in terms of the number of stocks used and the percentage of waste for each sample group. (The "Waste (%)" values in the table were calculated using "(Total Waste Length) / (Total Used Length)".

Prob.	Req. Item	Proposed Method		Tanır et al. (2018)		EP		Real Cut 1D	
		Stock Qty.	Wastege (%)	Stock Qty.	Wastege(%)	Stock Qty.	Wastege (%)	Stock Qty.	Wastege (%)
1a	20	9	2,44	9	2,44	9	2,44	9	2,44
2a	50	23	3,92	23	3,92	23	3,92	23	3,92
3a	60	15	0	16	6,67	15	0	15	0
4a	60	19	2,37	19	2,37	19	2,37	19	2,37
5a	126	53	5,29	53	5,29	54	7,28	53	5,29
6a	200	80	2,82	79	1,54	82	5,40	80	2,82
7a	200	68	1,04	69	2,53	69	2,53	68	1,04
8a	200	148	4,05	144	1,24	149	4,76	143	0,54
9a	400	150	0,80	151	1,47	155	4,16	150	0,80
10a	600	216	0,50	217	0,97	224	4,23	216	0,50

Table 2. Comparison of Results Obtained from Different Methods

The validation and comparative studies conducted have shown that the proposed solution method produces efficient results. Upon examining the table provided, it can be seen that the proposed solution method gives only slightly worse results than Real Cut 1D, a paid software, in the 8a sample group. The heuristic solution method suggested by Tanir et al. provided good results in the 6a and 8a sample groups, while the proposed solution method produced better results in the 3a, 7a, 9a, and 10a sample groups. Upon comparing the proposed solution method with the evolutionary algorithm proposed by Liang et al., it can be seen that the proposed method either produced the same or better results in almost all sample groups than the evolutionary algorithm.

The chart given below provides a clear visual representation of the values presented in the table, making it easier to comprehend and compare the results obtained from solution method with other methods, namely Tanir et al.'s heuristic method, Liang et al.'s evolutionary algorithm, and the commercial software Real Cut 1D. The results are presented for ten different sample groups, labeled from 1a to 10a. The horizontal axis represents the sample groups, while the vertical axis represents the average waste percentage. The proposed solution method is represented with a circle, Tanir et al.'s intuitive method with a square, Liang et al.'s evolutionary algorithm with a triangle, and Real Cut 1D with an equilateral quadrangle.

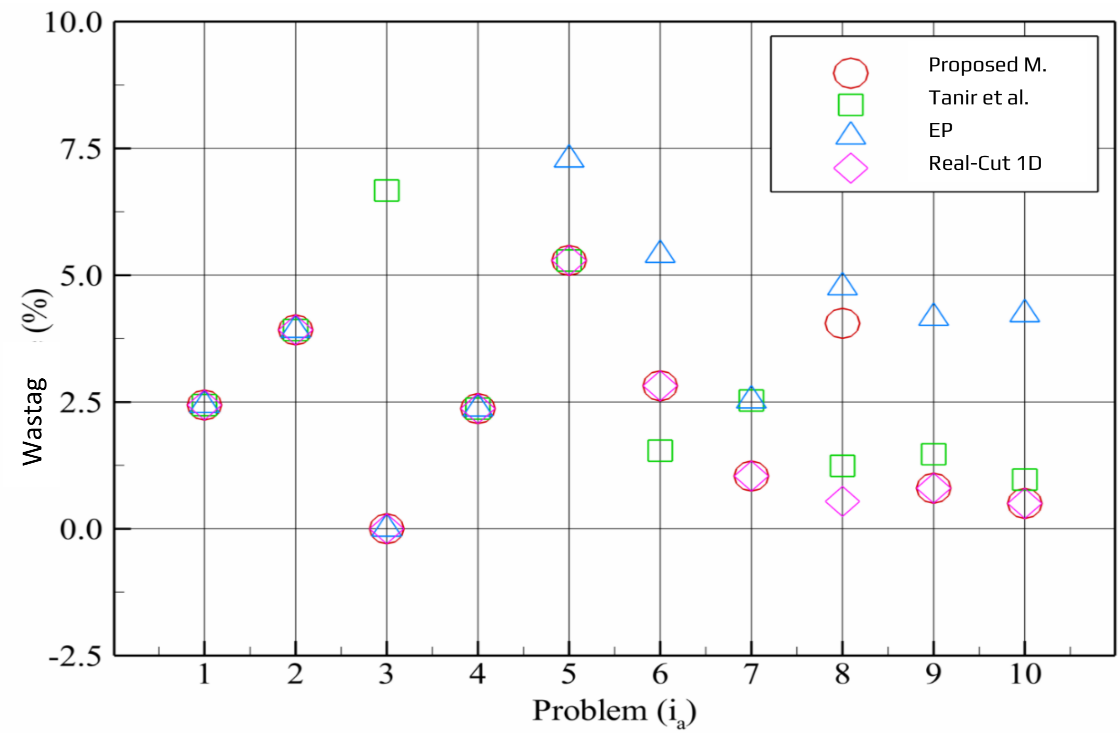

Figure 2. Comparison of Average Waste Percentage for Different Methods Across Sample Groups

To deepen the comparison and evaluate the effectiveness of the methods, the relative distance values were calculated with respect to the optimal solution for each problem group. (The relative distance formula is a percentage value obtained by subtracting the number of stocks used in the current solution from the number of stocks used in the optimal solution, and then dividing the result by the number of stocks used in the optimal solution) Relative distance values are a commonly used metric to evaluate the effectiveness of optimization algorithms. By comparing the number of stocks used in a particular algorithm's solution to the number of stocks used in the optimal solution, we can determine how close the algorithm's solution is to the best possible solution. This allows us to evaluate the effectiveness of different optimization methods in a quantitative manner. Furthermore, the use of relative distance values allows us to quantify the degree of deviation from the optimal solution. By expressing the distance as a percentage of the number of stocks used in the optimal solution, we can easily compare the degree of deviation across different problem sets and algorithms. This makes it easier to identify which algorithms are performing well and which ones may need improvement. The distance values relative to each algorithm's optimal solution are given in Table 3. The method with a relative distance value of 0 is the method that provides the most suitable solution in the relevant sample group. The values marked with an asterisk in the table express the percentage of the distance from the optimal solution for the relevant method in the applied problem dataset.

Problem	Proposed Method	Tanir et al. (2018)	EP	Real Cut1D
1a	0.000	0.000	0.000	0.000
2a	0.000	0.000	0.000	0.000
3a	0.000	0.067*	0.000	0.000
4a	0.000	0.000	0.000	0.000
5a	0.000	0.000	0.019*	0.000
6a	0.013*	0.000	0.038*	0.013*
7a	0.000	0.015*	0.015*	0.000
8a	0.035*	0.007*	0.042*	0.000
9a	0.000	0.007*	0.033*	0.000
10a	0.000	0.005*	0.037*	0.000

Table 3. Relative Distance Values for the Algorithms Compared to the Optimal Solution

It can be seen that the proposed solution method achieves the optimal solution in eight sample groups, and only fails to achieve the optimal solution in problems 6a and 8a. However, it is noteworthy that it provides the second-best result in problem 6a and a better result than the evolutionary algorithm in problem 8a. The graph displaying the relative distance is provided below to facilitate a better understanding of the comparison.

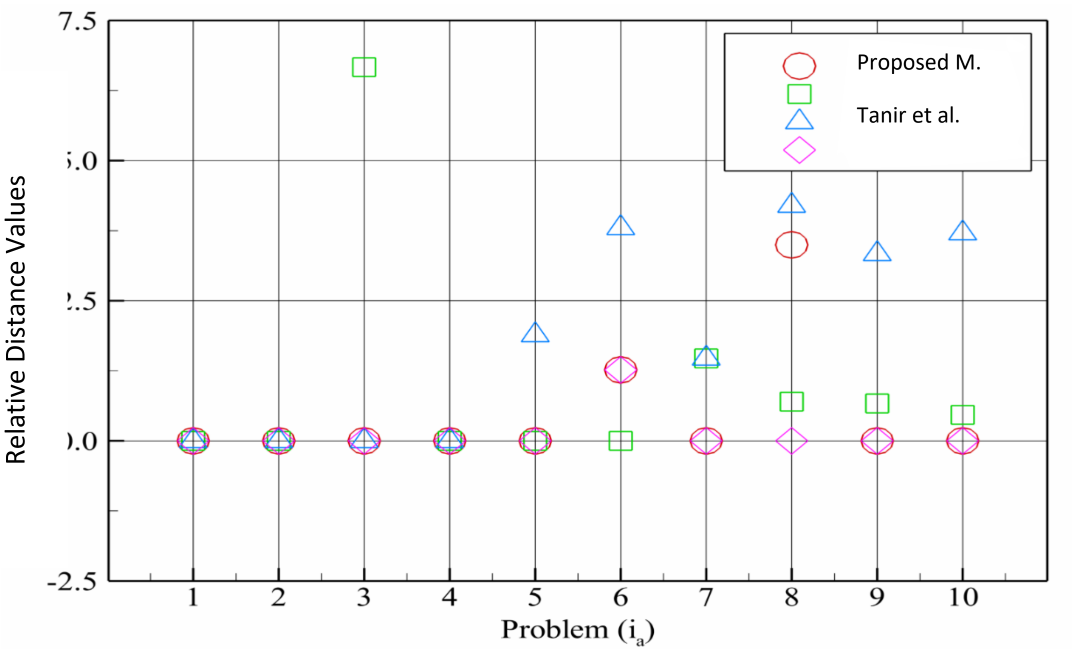

Figure 3. Performance Comparison of the Algorithms Based on Relative Distance Values

As can be clearly seen from the graph, the solution steps that were applied have produced successful results, indicating that the proposed method is a competitive and efficient approach. Among all the methods tested, Real-Cut 1D, which is a commercial and paid software, generally provides the most effective results. However, it is important to note that the proposed solution method only produced worse results than the software in one sample group. Overall, the proposed method proved to be a satisfactory and competitive approach, as evidenced by its performance in the majority of the tested sample groups.

5. Conclusion

The proposed solution method for the cutting stock problem presented in this study is a hybrid algorithm that combines two different techniques, namely the iterative trimm loss algorithm and Monte Carlo simulation. The first stage of the method utilizes an iterative trim loss algorithm to minimize the amount of waste associated with each stock, while efficiently assigning the cutting patterns.The second stage then utilizes Monte Carlo simulation to optimize the cutting plan further. By combining these two methods, the proposed solution method is able to produce a highly efficient cutting plan with minimum waste.

The effectiveness of the proposed hybrid solution method was validated through a series of comparative studies using 10 sample datasets. Our findings demonstrated that our solution method consistently generated efficient results, with waste percentages lower than or comparable to those obtained by other methods for nearly all sample groups.

Although Real Cut 1D outperformed the proposed solution method in the 8a sample group by a small margin, the proposed method demonstrated comparable performance to Real Cut 1D across all other sample groups. The heuristic method proposed by Tanir et al. provided good results in the 6a and 8a sample groups, but the proposed hybrid solution method produced better results in other sample groups.

Additionally, the proposed solution method outperformed the evolutionary algorithm proposed by Liang et al. in most sample groups.

The relative distance values calculated for the optimum solution of each algorithm provided additional evidence of the effectiveness of the proposed hybrid solution method. The findings indicate that the proposed method is a dependable and efficient approach to optimizing the cutting plan and reducing waste, which can potentially benefit manufacturing and production processes.

Further research could focus on improving the proposed method by incorporating other optimization techniques or refining the Monte Carlo simulation process. Additionally, applying the proposed method to different manufacturing and production contexts could provide valuable insights into its effectiveness and applicability.

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