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RESEARCH ARTICLE

Analysis of Queue Models in Simulation Applications

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ABSTRACT
With the advancement of technology, the speed and efficiency of information processing have become vital for meeting the growing demands of individuals and organizations. As time constraints increase, rapid and accurate access to information has gained critical importance. To address these challenges, organizations in the business and public sectors are increasingly relying on simulation methods, a core area of computer science, to optimize their responses to customer demands. Simulation provides a robust framework for analyzing and modeling complex systems. Within this framework, queue theory plays a central role by examining how systems handle incoming demands and offering insights into improving resource utilization, minimizing waiting times, and enhancing overall performance, particularly in service industries.
This study provides a detailed analysis of queue theory, exploring its fundamental principles, key features, and various models. Additionally, a comparative evaluation of different queueing models is conducted through simulation, assessing key performance metrics such as server utilization, maximum queue length, and average response time. The results indicate that model selection significantly impacts system efficiency, with certain models exhibiting superior performance under specific conditions. These insights equip organizations with the tools to develop more effective strategies, optimize their processes, and enhance responsiveness to evolving demands.

Keywords: Simulation, Queue theory, Performance metrics, Resource utilization, System modeling

1. Introduction

In today's rapidly evolving and highly connected world, waiting has become unavoidable, impacting individuals and organizations across various domains. Whether it is a customer waiting in line to order coffee, a patient waiting for their appointment at a hospital, or vehicles delayed at traffic lights, waiting is a universal phenomenon. As populations grow and societal demands become increasingly complex, businesses, public institutions, and other organizations face significant challenges in managing waiting times efficiently while maintaining high-quality service. The consequences of waiting extend beyond mere inconvenience. Excessive delays can lead to customer dissatisfaction, loss of business opportunities, and reduced operational efficiency. These inefficiencies in critical sectors such as healthcare, transportation, and logistics can also have broader societal and economic implications. Therefore, understanding and optimizing the dynamics of waiting systems is essential for enhancing service quality, resource utilization, and overall system performance.

Queue theory, a branch of operations research and applied mathematics, provides a robust framework for analyzing and managing waiting lines in various systems. By modeling the behavior of customers, service mechanisms, and system capacity, queue theory offers valuable insights into how organizations can minimize waiting times, improve resource allocation, and enhance customer satisfaction. From service industries like banking and retail to technical applications in computer networks and telecommunications, queue theory is critical in addressing the challenges posed by increasing demand and limited resources. Simulation, as a complementary tool, further enhances the practical application of queue theory. Using computational models, simulation allows organizations to mimic real-world processes, test different scenarios, and evaluate the effectiveness of strategies in a controlled environment. This combined approach enables data-driven decision-making, empowering organizations to develop efficient and adaptable solutions.

This study delves into the fundamental principles, characteristics, and applications of queue theory, particularly emphasizing its integration into simulation studies. By exploring performance metrics and real-world applications, the study aims to highlight the critical role of queue theory in improving organizational processes, meeting customer demands, and addressing

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the complexities of modern systems. As industries continue to evolve, the insights provided by queue theory and simulation remain indispensable for ensuring sustainability and competitiveness.

2. Literature Review

Queue theory, a fundamental branch of operations research, has been extensively studied across various disciplines due to its critical role in analyzing and optimizing waiting systems. From its theoretical foundations to practical applications, queue theory provides valuable insights into system performance, resource utilization, and customer satisfaction. Numerous studies have explored its use in diverse fields such as healthcare, transportation, telecommunications, and service industries, demonstrating its versatility and effectiveness. This section reviews the existing literature on queue theory, highlighting key contributions, practical implementations, and emerging challenges. Particular attention is given to integrating queue theory with simulation techniques, which offer enhanced capabilities for addressing dynamic and complex real-world systems.

Sztrik aimed to establish a technical foundation by addressing the subject of queue theory with its fundamental aspects [1]. Ulaş conducted a theoretical examination addressing a parallel-channel queue system by using the birth-death process to determine the parameters of a parallel queue system. Additionally, a two-heterogeneous-channel queue system was analyzed using a Poisson process [2]. Şimşek implemented a single-service channel model, examining the passages of tankers and other ships through the Istanbul Strait. The results indicated intense tanker traffic in the Istanbul Strait, with an increasing number of passing tankers [3]. Kiremitçi aimed to efficiently create transportation plans for ships and achieve cost savings by addressing the vehicle route planning problem in the maritime transportation sector [4]. Parlak focused on queue systems in healthcare institutions and the Central Physician Appointment System (MHRS). The study analyzes the effectiveness of appointment systems, information level, satisfaction level, and the benefits they provide patients accessing healthcare services [5].

Batur provided detailed information on the functioning of the evolving air transportation sector and aimed to offer solutions to the problems arising in this industry. The research included examples from Türkiye and worldwide air transportation [6]. Chaves conducted a study on two-stage queue models developed for the aviation sector. A single-channel multi-phase queue model was examined in the first stage, and in the second stage, a general distributed single-phase model was proposed [7]. Adan and Resing examined queue model examples in detail, providing technical information about mathematical modeling [8]. Ertuğrul and others studied queue theory and analyzed customer waiting queues at branches of two banks operating in the city of Denizli using queue theory [9]. Majid and Manoharan compared the M/M/c queue model from two different perspectives, comparing the model they developed [10]. Yıldız and Arslan examined students' waiting queues in the Central Cafeteria of Düzce University, calculating the average performance of the system [11]. Maragathasundari and others studied non-Markovian queue models for aircraft control systems [12].

Lan and Tang performed stability analysis by evaluating the probability of problem occurrence in the Geo/Geo/1 queue system [13]. Kim and others created a simulation-based queue model for unmanned and automatic systems by examining potential airport queue models [14]. Anosike and Nneka evaluated the adequacy of the Nigeria Nnamdi Azikiwe International Airport (NAIA) for current demand by mathematically modeling the passenger queue problem [15]. Smith analyzed M/M grouped queue models, considering rounding errors in numerical calculation situations [16]. Jawab and others aimed to optimize and enhance the passenger queue model at Fez-Sais Airport in Morocco [17]. Girginer and Şahin investigated the waiting queue problem during the use of sports equipment in sports facility operations through simulation methods. The study simulated the system in a sports facility using data collected over 45 days to identify factors affecting capacity issues [18]. Kumar and others studied the performance parameters of the M/M/1/N feedback customer queue [19].

Artalejo and Falin analyzed the renewable queue model, where a customer cannot receive service based on limited capacity, density, and other reasons. They compared M/G/1 and M/M/c queue models [20]. Smith and others examined a queue system in the M/M/N/N format where two types of users, prioritized and non-prioritized, attempt to reach N resources. This study aims to model future portable radio systems [21]. Ibe and Isijola examined a queue system with multiple vacations following the busy period in the M/M/1 queue model. They interpreted the differences between the model with zero duration, where no customer is served after the busy period, and the model with nonzero duration, where service can be provided immediately after the busy period [22]. Vandaele and others examined a traffic flow analysis traditionally based on experimental (empirical) methods and developed an analytical queue model to perform this analysis. They described the model established for traffic control and density analysis on a main road [23]. Using Bessel functions and probability techniques, Kumar examined the Markovian multi-phase queue system (M/M/c). According to the results, the established model could play a significant role in systems such as emergencies in hospitals and call centers [24].

Wang and Zhu proposed a dynamic queue model solved using the multiple-server model for excessive demand. They used an assumption involving customers joining the queue early or late, differentiating evaluation and cost criteria. They examined this model in systems such as shopping malls, restaurants, and highways [25]. Sharma and others explained the fundamentals and usage of queue theory, offering information on mathematical modeling [26]. Christien and others studied the sequencing of landing aircraft with different operational procedures in the three major ports of Europe. This study aimed to shed light on solving complex operations during peak hours [27]. Mehri and others worked on explaining basic queue models and focused on mathematical modeling [28]. Chew worked on a new modeling of the standard M/M/1 queue structure, proposing a queueing model in this new structure that includes single-phase and real/virtual customer types. They compared this model to the standard model with simulation support [29]. In his research book, Winston focused on the fundamentals of queue theory and mathematical modeling [30].

Awasthi studied the M/M/1/K finite capacity queue model on customer behaviors, including balking (customer leaving if the queue is too long) and reneging (customer leaving if the queue is moving too slowly) [31]. Som and Seth developed a Markov queue system for single-phase and finite arrivals. Addressing the encouraged arrival model, they created a model reflecting the effects of discounts and attractive offers implemented by companies. They examined the model numerically and through simulation [32]. Jhala and Bhatwala proposed a model to reduce airport queues and increase customer satisfaction [33]. Çevik and Yazgan developed a queueing model to determine customer waiting times in a bank and calculated the average efficiency of the system [34]. Poongodi and Muthulakshmi proposed a control chart method for the infinite capacity M/M/s queue model. This method aims to predict possible waiting times, maximum waiting times, and minimum waiting times in advance, considering customer satisfaction [35]. Using the operator analytics technique, Massey designed an M/M/1 Markov queue model for non-static situations. Using a common parameter to determine arrival and service rates, this technique reveals dynamic asymptotic behavior different from broad time interval analysis [36]. Idris and others analyzed data to determine flow constraints delaying departure operations at Boston Logan International Airport, comparing it with other airports [37].

Shone and others worked on the optimal control and modeling of aircraft queues at runway thresholds, providing a literature review of related studies [38]. Tiwari and others studied the M/M/1/N queue model with Poisson arrivals and exponential service times, researching the expected total minimum cost [39]. Thiagarag and Seshaiah worked on the landing aircraft queues, exploring the limits of analytical approaches and making inferences about how simulation methods should be used [40]. Bertsimas and Nakazato examined the MGEL/MGEM/1 queue model using the MGE method, a subclass of the Erlang distribution. They calculated the queue length distribution and waiting times using the first-come, first-served principle [41]. Karapetyan and others worked on the pre-departure sequencing of aircraft, conducting a study on the goals, requirements, and real-time decision mechanism of the system developed using an algorithm [42]. Aydın worked on determining the landing order and times of aircraft in the air. The study suggested that the aircraft scheduling problem could be solved using metaheuristic methods, achieving an optimal solution, and shedding light on future research related to ACP [43]. Arslan focused on efficiently using aircraft gates and optimizing the gate assignment process by considering factors that could affect passenger satisfaction [44].

Doğan worked on minimizing costs arising from unexpected flight routes by airline companies. For this purpose, they developed two different decision support systems by analyzing meteorological data retrospectively and making future predictions [45]. Fatima and others present the efficiency of patient management in healthcare institutions by comparing traditional queuing systems with modern technological advancements. The study highlights how integrating innovative technologies can improve operational efficiency, minimize delays, and enhance patient satisfaction [46]. Anita and others present a Markovian two-stage tandem queueing system with retrial policy and server vacation, where customers undergo service at both stations and those unable to be served immediately retry after a random time. The system's performance is analyzed through birth-death balance equations, and the effects of various parameters are illustrated graphically [47]. Dhibar and Jain analyzed a Markovian retrial queueing system with two types of customers, unreliable servers, and Bernoulli feedback, focusing on customer decisions to join or balk based on service profit and delay cost. The study employs Chapman-Kolmogorov equations and the probability-generating function method to derive performance metrics, and optimization techniques like PSO and GWO are used for cost optimization and QoS enhancement, with results validated through numerical simulations [48].

Amjath and others review past research on the performance evaluation and optimization of Material Handling Systems (MHSs) using queueing network models. It comprehensively analyzes relevant research questions and adopts systematic literature review, bibliometric, and content analysis techniques to offer insights for scholars and practitioners in material logistics [49]. Ambika and others examine a queueing model in production management with working vacations and Bernoulli vacations, where the manufacturing unit operates at a reduced rate during maintenance phases. Mathematical techniques are used to calculate transient state probabilities, and numerical examples illustrate the impact of these dynamics on production management [50]. Çakmak and Torun evaluate the performance of different queue management algorithms in LTE networks through simulations conducted in the NS-3 environment [51]. Çakmak and Albayrak comprehensively analyze various active queue management techniques used in mobile communication networks. It examines different algorithms, their working principles, and their impact on network performance [52]. In another work, they also analyze the performance of queue management algorithms between the Remote-Host and PG-W in LTE networks [53]. Gündoğar and others analyze a spring mattress manufacturing line to identify and eliminate bottlenecks using the Theory of Constraints (TOC). By applying simulation-based methods in Arena 13.5, they tested various scenarios to optimize production flow [54].

Overall, the reviewed literature demonstrates the broad applicability of queueing models across various domains, highlighting their potential to enhance operational efficiency, minimize waiting times, and improve customer satisfaction. Similarly, our study compared simulations of different queueing models to evaluate their performance under various conditions. By analyzing models such as M/M/1, M/D/1, and others, we aimed to provide deeper insights into their practical applications and identify which models most effectively optimize service systems. This comparative analysis offers valuable contributions to understanding queueing theory and its role in improving operational processes across diverse industries.

3. Queueing Theory

3.1. What is Queueing Theory?

Queueing theory is a branch of applied mathematics and operations research that analyzes waiting lines or queues. It studies the behavior of customers arriving for service, the processes they undergo, and the factors affecting system efficiency. By examining these dynamics, queueing theory provides valuable insights into optimizing service processes and minimizing delays. At its core (Figure 1), queueing theory evaluates key components such as arrival rates (the frequency of customer arrivals), service rates (the speed at which services are provided), and queue disciplines (rules governing the order of service). Ordinary queue disciplines include First-In-First-Out (FIFO), Last-In-First-Out (LIFO), and priority-based approaches, each suited to different operational contexts [55].

For instance, FIFO is often used in retail checkout lines, while priority-based systems are common in emergency healthcare services. This theory is not only concerned with the mathematical analysis of queues but also with their practical implications. By modeling waiting systems, organizations can improve resource utilization, reduce waiting times, and enhance customer satisfaction. The queueing theory finds applications in diverse fields, including banking, transportation, telecommunications, and healthcare, making it an essential tool for academic research and practical decision-making. As modern systems grow more complex, queueing theory is increasingly combined with simulation techniques to address dynamic and unpredictable scenarios. This integration enables organizations to test and refine strategies in virtual environments, ensuring optimal performance in real-world applications.

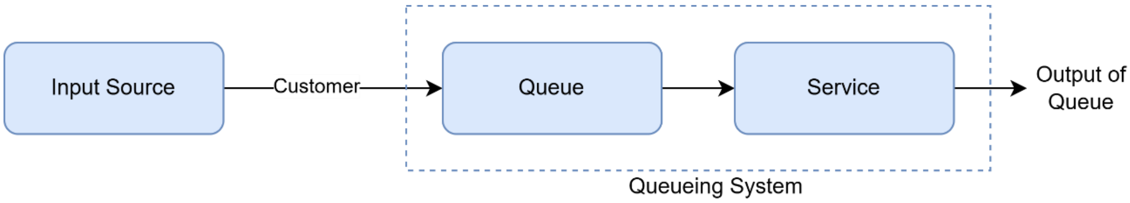

Figure 1. Basic Queueing System Schematic

3.2 Poisson Process

Let $\{ N(t): t \geq 0\}$ be a counting process and $\lambda > 0$. A counting process satisfying the following properties is called a Poisson process with rate λ [56]:

(Property 1) Independent Increments: The process has independent increments. The number of events appearing in non-overlapping time intervals is independent of each other. For any ordered time, indices $0 \leq t_1 < \cdots < t_n$, the random variables $N_{t_1}, N_{t_2} - N_{t_1}, \ldots, N_{t_n} - N_{t_{n-1}}$ are independent.

(Property 2) Poisson Distribution: The number of events occurring within a unit time interval follows a Poisson distribution with an average rate of λ. Additionally, the number of events appearing within a time interval of length t follows a Poisson distribution with a mean of λt:

$$\Pr(N_{t+s} - N_s = k) = \frac{e^{-\lambda t}(\lambda t)^k}{k!}, k = 0,1,2, \ldots \tag{1}$$

(Property 3) Considering a small positive real number $h \geq 0$: the possibility of a single event appearing in the time period $[t, t+h)$ is:

$$\Pr(N_{t+h} - N_t = 1) = \lambda h + o(h) \tag{2}$$

On the other hand, the probability of at least two events occurring in $[t, t+h)$ is:

$$\Pr(N_{t+h} - N_t \geq 2) = \Pr(N_h \geq 2) = o(h) \tag{3}$$

The probability of no events occurring in $[t, t+h)$ is:

$$\Pr(N_h = 0) = 1 - \lambda h - o(h) \tag{4}$$

These probability equations imply that the likelihood of a substantial number of events happening in small time intervals tends to be small.

Theorem: If N_t has a Poisson distribution and X_n represents the time among the (n−1)th and nth events, then the inter-arrival times are independent and follow an exponential distribution with a mean of $1/\lambda$ [57].

3.3. Characteristics of Queueing Theory

Fundamentally, a queue consists of two main components:

- The side requests the service (also known as the arrival side or customers).
- The side provides or completes the service (also known as the service side or server).

The elements of the queue system are further detailed in the following subsections.

3.3.1. Called Population

The population of possible customers is known as the calling population. Although the number of potential customers is technically finite, it is often assumed to be infinite to simplify the model. This assumption is reasonable when the potential customer base is large, particularly if the number of customers currently receiving or waiting for service represents only a slight fraction of the total population. Assuming an infinite population implies that the arrival rate of customers is unaffected by the number of customers already in the system, allowing the arrival rate to remain constant over time [58].

3.3.2. System Capacity

Queue systems often limit the number of customers occupying the waiting line or the system. If the system reaches capacity, incoming customers cannot enter and are immediately redirected to the calling population. However, some systems are designed with infinite capacity, allowing unlimited customers to enter. In systems with restrained capability, a distinction is made between the arrival rate, the number of customers arriving per unit time, and the effective arrival rate, which represents the number of customers per unit time that enter the system [59].

3.3.3. Arrival Process

For infinite population models, the arrival process is characterized by the time intervals between consecutive customer arrivals. Arrivals can occur at planned or random times. In the case of random times, the intervals between arrivals are typically characterized by probability distribution. Customers can also arrive individually or in groups, with the party being of a fixed or random size. For random arrivals, the Poisson arrival process is the most significant model and the primary focus of our consideration. Let A_N, represent the inter-arrival time for the N, then between the $(N-1)_{th}$ and N_{th} customer, it follows an exponential distribution with an average of $(1/\lambda)$ per unit of time. The arrival rate is $\lambda, which$ denotes the average number of customers arriving per unit of time. Over a long time, interval T, the total number of arrivals follows a Poisson distribution with an average of λT customers [60].

3.3.4. Queue Behavior and Queue Discipline

Queue performance refers to customers' actions and decisions while waiting for service. During this waiting period, customers may sometimes decide not to join the queue, which is known as balking. Customers who have joined the queue might leave before receiving service, which is called reneging. Additionally, when multiple queues are available, customers might switch to a different queue if they perceive it to be moving faster, a behavior called jockeying. Queue discipline refers to the rules determining the order in which customers are selected for service. One of the most ordinary queue disciplines is FIFO, where the customer who has been waiting for the longest is served first. Another less common discipline is LIFO, where the most recent customer who joined the queue is served first. Service-In-Random-Order (SIRO) involves selecting customers for service randomly without regard to their arrival time or position in the queue. In some systems, customers may be served based on priority. In such cases, customers with higher priority (e.g., VIPs or urgent cases) are served before others, regardless of their position in the queue [61].

3.3.5. Service Times and Service Mechanism

The service times for consecutive arrivals are denoted as $S_1, S_2, ...$ can either be constant or random. In cases where they are random, $\{S_1, S_2, S_3, ...\}$ is typically modeled as a sequence of independent and identically distributed random variables. While service times for customers of the same type, class, or priority often share the same distribution, customers of several types may have distinct service time distributions. Furthermore, service times in some systems may vary based on factors such as the time of day or the length of the waiting line. A queue system comprises a network of service counters and interconnected queues. Each service center contains a certain number of servers, denoted b C, operating in parallel. When a customer reaches the front of the queue, they are assigned to the first available server. The parallel service mechanism can take different forms: a single server $(C = 1)$ or multiple servers $(1 < C < \infty)$[62].

4. Analysis of Queue Models in Simulation Applications

4.1. Representation of Queue Models

There are numerous queue representations, and a six-character notation is commonly used to represent these models. The first three characters of this notation were proposed by Kendall in 1953 and signify the arrival distribution, service time distribution, and the number of servers (channels). A. M. Lee later added the fourth and fifth characters in 1966. In 1968, Hamdy A. Taha defined the last character. This notation is often used in software to describe and define queue models. The Kendall notation summarizes three key factors: arrival distribution, service time distribution, and the number of servers, represented as A/B/C/D/E/F. These characters correspond to the following [63]:

- A: Distribution of arrivals
- B: Distribution of service times
- C: Total parallel servers (channels)
- D: Queue rules
- E: System capability
- F: Population size,

Standard notations that can replace A and B include M (exponential), D (constant or deterministic), E_k (Erlang), and G (general). These notations represent the characteristic distribution type for arrivals and service times.

4.2. The Importance of Queue Theory for Simulation

Queue theory is a crucial concept in simulation studies, especially when simulations assess and enhance the performance of businesses, organizations, or systems. The following aspects are of particular significance in highlighting the importance of queueing theory in the context of simulation [64]:

1. Analysis of Waiting Times: Queue theory is utilized to understand how waiting and queues form within a system. When used to model a specific process or point in a system during a simulation, it helps evaluate how waiting occurs and assesses its impact on process efficiency.
2. Optimization of Resource Utilization: Queue theory analyzes the effective utilization of resources such as personnel, machinery, service points, etc. It can be used in simulation models to develop strategies for enhancing the efficiency of specific resources or service points and optimizing overall capacity.
3. Improvement of Service Quality: Queue theory is valuable for evaluating and improving the service quality of a system. Since waiting times directly impact customer satisfaction, using queue theory through simulations allows for optimizing service processes, ultimately enhancing the overall customer experience.
4. Determination of Performance Metrics: In simulation models, queue theory can be used to identify specific performance metrics. These metrics, such as average waiting time and resource utilization efficiency, can be determined to assess and improve system efficiency.
5. Risk Analysis: Queue theory applies to understanding how a system behaves in certain scenarios and evaluating potential risks. Employing queue theory through simulations provides insights into how a system would respond in specific scenarios.

In conclusion, queue theory is a powerful tool for analyzing systems' effectiveness, performance, and resource utilization within simulation models. This theory can assist businesses in optimizing their processes, making more efficient use of resources, and ultimately improving customer satisfaction.

4.3. Applications of Queue Theory

Queue theory has diverse applications across multiple sectors, which is crucial in optimizing processes and improving efficiency. Some key areas where queue theory is commonly applied include [65]:

1. Service Sector: In service industries such as banks, hospitals, and restaurants, queue theory is employed to analyze and optimize customer service processes. It reduces waiting times, improves service quality, and manages staff capacity.
2. Transportation and Logistics: Queue theory is applied in transportation and logistics sectors, including traffic management, airport flight scheduling, and bus terminal operations. It is used to evaluate the effectiveness and efficiency of transportation systems, aiming to reduce waiting times and develop strategies for more effective resource utilization.

3. Telecommunications: Queue theory is utilized in telecommunication systems such as call centers, data transmission lines, and internet service providers. It helps evaluate network performance, optimize capacity, and enhance service quality.
4. Manufacturing and Industrial Processes: In industrial settings, including manufacturing lines, inventory management, and order processing, queue theory is applied to improve production efficiency and optimize material flow.
5. Computer Science: Queue theory is used in computer science fields such as computer networks, data transfer between processors, and database management. It helps evaluate system performance and response times.
6. Finance and Banking: Queue theory finds application in areas like bank queues, ATMs, and the processing of financial transactions. It aims to improve customer service, minimize waiting times, and increase transaction capacity.
7. Healthcare Services: In healthcare, queue theory is applied to emergency services, appointment scheduling, and treatment processes. Its goal is to reduce patient waiting times and assist in the more effective management of healthcare services.

These application areas highlight the versatility of queueing theory as a valuable tool for optimizing system performance across various industries. For instance, queuing theory can be employed in the banking sector to optimize customer service operations. Banks can adjust staffing levels by analyzing factors such as waiting times and transaction volumes to ensure customers are served more efficiently, particularly during peak hours. This reduces waiting times, improves customer satisfaction, and more effective resource utilization. In logistics, queuing models can enhance material handling and warehouse operations. Logistics companies can reduce bottlenecks, minimize delays, and improve throughput by optimizing the flow of goods and managing inventory more effectively. Queuing theory in this context contributes to better scheduling of tasks and resource allocation, leading to more streamlined operations and cost savings. By applying queuing theory in these real-world settings, organizations can achieve tangible benefits such as increased operational efficiency, reduced costs, and improved customer satisfaction, ultimately fostering a more competitive and sustainable business model.

4.4. Queue Theory Performance Metrics

Various sources may use various terms to describe the metrics used to evaluate the performance of queue models, but these metrics essentially measure the same core concepts. Waiting time refers to the duration customers spend waiting before receiving service. In contrast, the time spent on the system is the total time a customer spends, including waiting and service time. The distribution of the number of customers in the system reflects the number of customers present at any given time. Workload distribution is the total service time required for waiting customers and the remaining service time for the customer being served. The service station’s busy time refers to the continuous duration during which the service station remains occupied and operational. These metrics provide valuable insights into a queueing system's overall performance and efficiency [66].

Key performance metrics include average waiting time and time spent in the system. These metrics are significant in studies aimed at understanding system performance. The operational characteristics of steady-state queue systems can be calculated using various formulas, which help assess how the system performs under different conditions.:

- λ (Average Arrival Rate): The mean number of customers that arrive on the system within a unit of time.
- μ (Average Service Rate): The mean number of customers that serve in the system within a unit of time.
- p (Average System Utilization Rate): λ/μ, the mean system utilization rate.
- L (Average Number of Customers in the Queue System): $\lambda/(\mu - \lambda)$, The mean number of customers present in the queueing system at any given time.
- L_q (Average Number of Customers Waiting in the Queue): pL, The mean number of customers waiting in the queue for service at any given time.
- W (Average Time Spent in the System): $1/(\mu - \lambda)$, The total average time a customer spends in the system, encompassing both the waiting time and the service time.
- W_q (Average Time Spent Waiting in the Queue): pW, the average time spent on delay in the queue.
- P_n (Probability of having n Customers in the Queue): $(1 - p)p^n$, the possibility of having n customers in the queue at any given time.

These formulas and models should be used under the condition that the service rate is greater than the arrival rate ($\mu > \lambda$). Otherwise, the queue can excessively lengthen. Therefore, it is essential to ensure that this condition is met before using these formulas and models.

4.5. Performance Evaluation of Queueing Models

In this section, we compare several widely used queueing models based on three key performance indicators: Server Utilization, Maximum Queue Length (MQL), and Average Response Time (ART). These parameters are essential for evaluating the efficiency and effectiveness of different queuing systems under varying conditions. The queueing models considered in this comparison include the following:

- M/M/1: Single-server queue with exponential inter-arrival and service times.
- M/D/1: Single-server queue with exponential inter-arrival times and deterministic service times.
- M/N/1: Single-server queue with exponential inter-arrival times and Normal-distributed service times.
- M/U/1: Single-server queue with exponential inter-arrival times and uniform service time distribution.
- M/Weibull/1: Single-server queue with exponential inter-arrival times and Weibull-distributed service times.
- M/LogNormal/1: Single-server queue with exponential inter-arrival times and LogNormal-distributed service times.
- M/Erlang/1: Single-server queue with exponential inter-arrival times and Erlang-distributed service times.

In queueing theory, the calculation of key performance metrics such as server utilization, ART, and MQL generally follows specific formulas. Still, the exact calculation depends on the type of queueing model and its respective characteristics. Server Utilization is typically calculated using the formula $\rho = \frac{\lambda}{\mu}$, where λ is the arrival rate and μ is the service rate. This formula holds for most queueing models, although slight variations may exist depending on the system's characteristics. ART is calculated based on Little's Law and specific model characteristics. For many models, the general formula is $W = \frac{1}{\mu-\lambda}$, with modifications made for different service distributions (e.g., Poisson, deterministic, Weibull). MQL is typically computed using $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$, but in models with capacity restrictions (such as M/N/1), this formula must be adjusted to account for the system's maximum capacity. Each queueing model (e.g., M/M/1, M/D/1, M/N/1) introduces specific variations in these formulas based on factors such as arrival rates, service rates, and the type of service distribution. Models with deterministic, uniform, or specialized distributions (e.g., Weibull or LogNormal) require tailored calculations that reflect these distribution characteristics. While general formulations exist, the exact metrics depend on the specific queueing model and its associated parameters.

The analysis uses a mean inter-arrival time of 4.5 minutes, representing the average time between successive arrivals. The mean service time, the average time a server takes to serve a customer, varies across three values: 2.5, 3.2, and 4. The Sigma value, representing the standard deviation of the service time distributions, is also set to 0.6. The distribution parameters are adjusted based on the mean service time for models involving non-exponential service times, such as Weibull, LogNormal, and Erlang. Specifically, for each distribution, the shape and scale parameters (for Weibull), the mean and sigma (for LogNormal), and the k and lambda values (for Erlang) are tailored to match the corresponding service time characteristics. The values for the mean service time (2.5, 3.2, and 4) and sigma (0.6) were chosen to represent different service scenarios with varying system utilization and congestion levels. The mean service time values allow for analyzing queueing performance under different operational conditions: 2.5 represents a relatively low service time, indicating a faster processing rate with lower congestion; 3.2 represents a moderate service time, balancing efficiency and queuing effects; and 4 represents a higher service time, simulating a more congested system with longer waiting times. These values ensure meaningful comparisons across different queueing models by examining their impact on key performance metrics such as server utilization, maximum queue length, and average response time under different conditions. The selected sigma value of 0.6 also introduces controlled variability in service times for models incorporating stochastic distributions, such as LogNormal and Weibull, ensuring realistic variations without extreme deviations. These parameters were carefully determined to comprehensively evaluate queueing behavior while maintaining system stability and producing interpretable results. To ensure high accuracy and minimize the impact of random fluctuations, the system is simulated with 1,000,000 arrivals. This large sample size guarantees that the results are statistically reliable and represent real-world conditions. The performance metrics, including server utilization, MQL, and ART, are computed for each model under these conditions. The results will provide valuable insights into the strengths and limitations of each queuing model, offering a basis for selecting the most appropriate model for different system requirements.

Figure 2 presents the ART for various queueing models at three mean service times: 2.5, 3.2, and 4 minutes. The comparison of ART across various queueing models reveals significant insights into the behavior of the systems under different service time distributions. The M/M/1 model consistently shows the highest ART, especially as the mean service time increases. This can be attributed to its exponential distribution, which leads to high arrival and service times variability, causing greater waiting times. In contrast, models with more predictable service time distributions, such as M/D/1, M/N/1, and M/U/1, exhibit lower response times. The M/D/1 model, with its deterministic service times, performs particularly well in maintaining stable response times. Models utilizing more flexible service time distributions, like M/Weibull/1 and M/LogNormal/1, show slightly higher response times than the deterministic models but still offer improvements over M/M/1. The Weibull and

LogNormal distributions capture more complex real-world service time behaviors, leading to more accurate performance predictions in certain systems. Lastly, the M/Erlang/1 model exhibits lower ART in most scenarios, demonstrating improved performance in handling service time variations. Its ability to achieve relatively lower response times suggests that it may offer advantages in systems where reducing waiting times is a priority. The results emphasize the importance of selecting an appropriate queueing model based on system characteristics. While M/M/1 provides a baseline for simple scenarios with exponentially distributed service times, models such as M/D/1, M/Weibull/1, and M/Erlang/1 show varying performance characteristics that may benefit systems with different service time distributions.

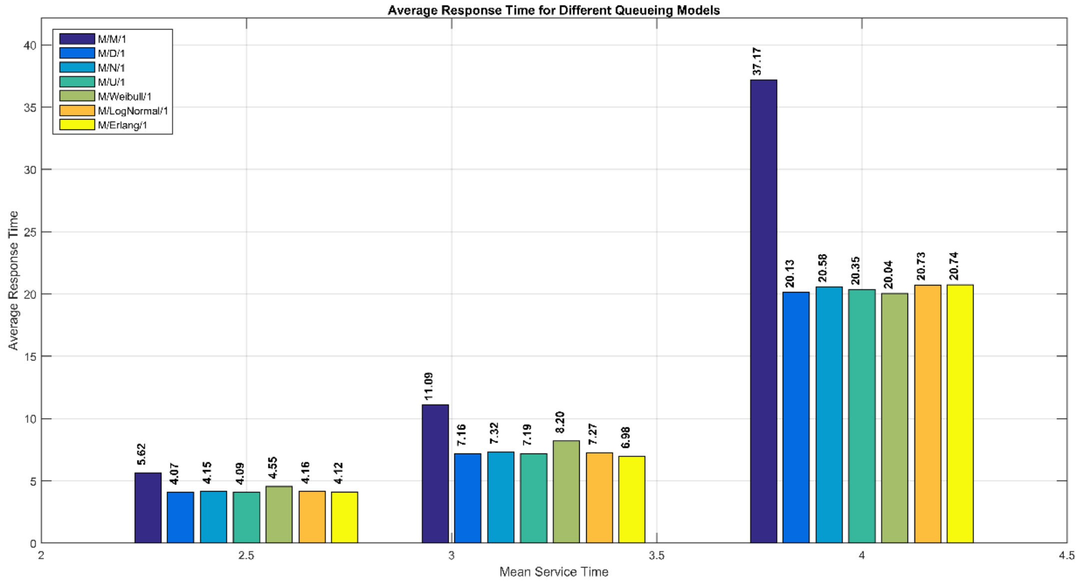

Figure 2. The Average Response Time of the Models

As the mean service time increases, the MQL in Figure 3 tends to grow for all models, reflecting the direct relationship between service time and queue length. However, the performance of different queue models varies. The M/M/1 model exhibits the highest MQL across all service times, with values of 21, 37, and 85, respectively. This is due to the variability introduced by the exponential distribution of service times, which leads to larger fluctuations and higher queue lengths. On the other hand, the M/D/1 model shows the best performance, with the lowest MQL at each mean service time value: 12, 20, and 42. This is expected due to the deterministic service times in the M/D/1 model, which result in more predictable and stable system behavior, reducing the chances of large queues building up. The M/N/1 and M/U/1 models display moderate increases in MQL as service time increases, but they still perform better than the M/M/1 model, with values of 13, 21, and 49 for M/N/1, and 15, 24, and 45 for M/U/1. These models exhibit better control over the queue than the M/M/1 model but are not as efficient as M/D/1. The M/Weibull/1 and M/LogNormal/1 models demonstrate moderate MQL values that are higher than M/D/1 but lower than M/M/1, showing that the flexibility of these distributions in capturing variability leads to slightly higher queue lengths (17, 26, 54 for M/Weibull/1 and 12, 21, 45 for M/LogNormal/1). Finally, the M/Erlang/1 model, while still relatively efficient in managing queue lengths, performs slightly worse than M/D/1, with MQL values of 13, 20, and 47. The results indicate that the Erlang distribution leads to shorter queue lengths than the M/M/1 and M/Weibull/1 models in the tested scenarios. This suggests that the choice of service time distribution plays a significant role in queue behavior, highlighting the need for careful model selection based on system requirements.

When analyzing the server utilization in Figure 4, values across different queue models, we observe that the values for all models are quite similar, particularly when the mean service time increases. This suggests that the models operate with relatively consistent utilization rates, with minimal differences between the deterministic and stochastic models. The M/M/1, M/D/1, M/N/1, M/U/1, M/LogNormal/1, and M/Erlang/1 models all show a steady increase in server utilization as the mean service time grows, with values progressing from 0.5552 to 0.7119, and then to 0.8871 as the service time reaches 4. The increased utilization across these models is expected because longer service times mean the server is busy for a greater portion of the time. However, the M/Weibull/1 model shows slightly lower utilization than the other models. At mean service times of 2.5, 3.2, and 4, the utilization values for M/Weibull/1 are 0.5234, 0.6694, and 0.8355, respectively. This is likely due to the shape of the Weibull distribution, which, depending on the shape parameter, can produce more variability in service times, leading to periods where the server is idle more often than in models with deterministic or less variable distributions.

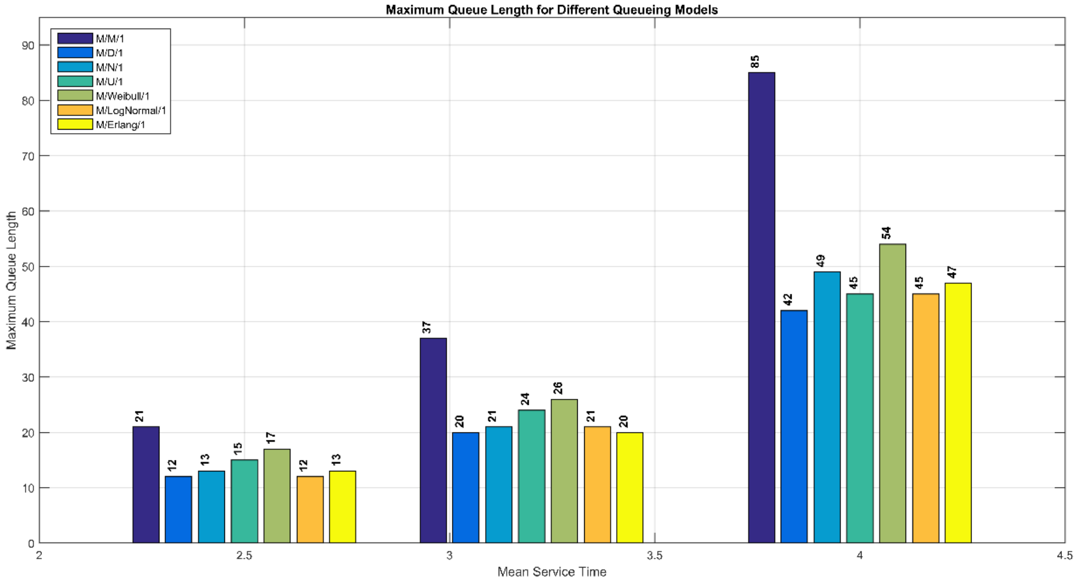

Figure 3. Maximum Queue Length of the Models

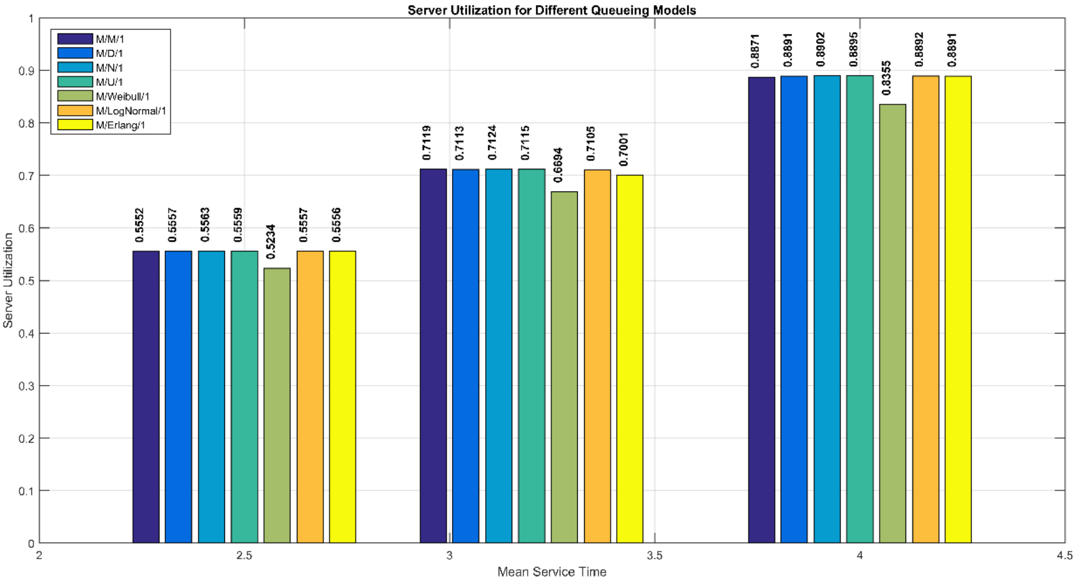

Figure 4. Server Utilization of the Models

5. Conclusions

This article comprehensively examines queueing theory and its role in simulation applications. It offers a detailed exploration of fundamental concepts, including input queues, output queues, service points, and waiting lines, laying a solid foundation for understanding the core elements of queueing theory. The study also delves into mathematical models for assessing processor speed, advanced queueing systems, and overall performance.

In addition to covering the basic principles of queueing theory, this article emphasizes its mathematical underpinnings, offering readers a well-rounded understanding of the subject. A simulation-based application compares the performance of different queueing models under varying service time conditions. The results reveal that models such as M/D/1 and M/Erlang/1 generally lead to shorter queue lengths and reduced average response times compared to M/M/1 in specific scenarios. This highlights the significance of choosing an appropriate model based on system requirements to optimize efficiency and performance.

In conclusion, this study demonstrates that queueing theory is a powerful tool for simulating and optimizing system performance when applied correctly. The findings underscore the importance of selecting suitable queueing models based on system characteristics, as different models exhibit distinct advantages in handling service time variations. By integrating theoretical insights with simulation-based analysis, this study provides a valuable resource for researchers and practitioners seeking to enhance system performance through queueing theory applications.

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Abdullah SEVİN contributed to the conception, design, writing, technical support, material support, and critical content review. Göktuğ YAMAN contributed to the conception, design, data collection, data analysis and interpretation, writing, technical support, material support, and literature review. Durdali ATILGAN contributed to the conception, design, writing, technical, and material support. Each author played a vital role in developing this work, ensuring its quality and accuracy.

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Conflict of Interest Notice

Authors declare that there is no conflict of interest regarding the publication of this paper.

Ethical Approval and Informed Consent

It is declared that scientific and ethical principles were followed during the preparation process of this study. All the studies referenced in this paper have been properly acknowledged in the bibliography. Ethical approval for this study was not required as it does not involve human participants or animals.

Plagiarism Statement

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